TOWARDS THE CLASSIFICATION OF NON-POLYNOMIAL INTEGRABLE EQUATIONS

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The classification of scalar polynomial evolution equations with fixed scaling weight has been obtained in [Sanders,Wang,1998], where it is shown that all scalar polynomial evolution equations with fixed scaling weight, of order higher than or equal to seven, are symmetries of an integrable equation of lower order. This result settles the long standing search of integrable hierarchies other than the Korteweg-deVries, Sawada-Kotera and Kaup equations in the class of polynomial scale invariant equations.

In this talk we shall discuss previous work and give some new results on the classification problem of integrable evolution equations for the non-polynomial case. The method the we use is the existence of "canonical conserved densities", which are the coefficients of the D^{-1} term in the formal series expansion of the recursion operator [Mikhailov, Shabat, Sokolov,1991]. For evolution equations of even order, all canonical densities are trivial; for the KdV hierarchy all canonical densities are non-trivial, while the Sawada-Kotera and Kaup hierarchies are characterized by the triviality of the canonical density ρ_2.

In previous work it has been first shown that arbitrary (non-polynomial) integrable evolution equations are quasilinear [Bilge,2005]; it is proved that scalar evolution equations of order m greater than or equal to 7 that are integrable in the sense of admitting nontrivial canonical conserved densities ρ_i of for i=-1 and i=1 are linear in u_m . Then it is proved that evolution equations admitting the conserved densities ρ_i for i=1 and 3 are polynomial in the variables u_m, u_{m-1} and u_{m-2} [Mizrahi, Bilge 2008]. Our initial guess was that all integrable equations would be polynomial, but for lower orders (order 5 up to 13) we have recently shown that integrable equations are polynomial in the derivatives higher than u_3 and the coefficient of the top order has the same form at every order. The classification of quasilinear evolution equations of order 5 is more or less complete: We have obtained a special solution fort he KdV class and the general solution for the Sawada-Kotera-Kaup class. These are technically new equations that are expected to be transformable to the standard ones but the transformations are not straightforward.