SUZUKI 2-GROUPS

Abstract

Suzuki 2-groups are studied: abelian of arbitrary exponent and nonabelian of exponent 4. For any Suzuki 2-group, one can associate a ground field which makes the theory of Suzuki 2-groups deeper. Let (G, T) be a Suzuki 2-group of exponent 2^n and I be the subgroup of involutions in G. We put $K = T/C_T(I) \cup \{0\}$ where 0 is a new symbol. Then we can define the multiplication on K by extending the group operation of $T/C_T(I)$ and addition on K by pulling back the group operation of I to K. Then K becomes a field where $I \rtimes T/C_T(I)$ is isomorphic to the affine group $K^+ \rtimes K^\star$ with K^+ and $K^\star$ are the additive and the multiplicative groups in K, respectively. Classification of a Suzuki 2-group (G, T), means determining the structure of the group G which admits such an action of T.

We proved uniqueness of an abelian Suzuki 2-group (G, T) of any given exponent 2^n over a perfect ground field K, by showing that G is isomorphic to the algebraic group $K \times ... \times K = K^n$ over K and G is an extension of the field K by K^{n-1}.

Then, we analyzed the role of "perfectness" assumption on the field, in case of exponent 4, we provide a classification of abelian Suzuki 2-groups of exponent 4 over an arbitrary field in terms of a certain cohomological invariant. We proved that there is a one-to-one correspondence between the family of abelian Suzuki 2-groups of exponent 4 over a field K of characteristic 2 and elements of a certain subset of the 2-dimensional cohomology group $H^2(K, K)$.

Nonabelian Suzuki 2-groups G of exponent 4 are classified into several types. One type appears when G is free over a perfect field K such that for any element $g \in G$, the subgroup $< g^T >$ is abelian. We call G a quasi-abelian Suzuki 2-group and give the classification in terms of a map $f : K \times K \to K$ satisfying certain properties. Another type of G, which we call smart Suzuki 2-group, is a nonabelian Suzuki 2-group of exponent 4 where T acts freely and transitively on G/I. In this case, we introduce a pair of fields K and k of characteristic 2 which we call the wide and the narrow fields associated to G, respectively. We describe the group structure in terms of the characteristic map $\alpha : K \to k$. We provide also some examples of nonabelian Suzuki 2-groups and give some criteria for the existence of their linear presentation by 3×3 matrices.